1 Extreme Events: Magic, Mysteries, and Challenges

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Summary. Extreme events (henceforth Xevents) occur in natural, technical and societal environments. They may be natural or anthropogenic in origin, or they can arise simply from "chance". They often entail loss of life and/or materials. They usually occur "by surprise" and therefore often only become the focus of scientific attention after their onset. Knowledge of Xevents is often rather fragmentary, and recorded experience is limited. Indeed, scientists do not really understand *what* causes extreme events, *how* they develop, and *when* and *where* they occur. In addition, we are rarely able to cope with their consequences, due to lack of anticipation and preparedness. All this has motivated us, the editors of this volume, to bring together specialists from a variety of fields of expertise, all of whom have a common background in mathematics and physics. We asked them to write their views about Xevents. The result is the present book of essays that will (hopefully) enable the reader to unlock the mysteries surrounding Xevents.

1.1 Why Study Xevents?

There is a long tradition of phenomenological studies of Xevents in human history. Let's focus on two examples. The first refers to the water levels of the Nile, which have been recorded for over 5000 years, providing a remarkable hydrological chronology of the lowest and highest water levels. The water level of the Nile has been discussed and analysed from ancient times in relation to religion, philosophy, and economics and human welfare: hunger when the water level sank to a minimum and disaster when the Nile was too high. Moderate flooding of the Nile delta, however, has been known to be beneficial to agriculture for many millennia.

The second example refers to earthquakes. Here, the chronology is not reflected in numbers, as in the case of the Nile, but mostly in oral or written history, poems, or newspaper articles. Records go back 3000 years, beginning with the Mt. Taishan earthquake in the Shandong Province of China. The Lisbon earthquake of November 1755 received much attention. It not only triggered earthquake research in Europe, but also served as the focus for various publications, ranging from Kant's essays about the causes of earthquakes to Voltaire's *Poème sur le désastre de Lisbonne.* What all of these reports have in common, however, is that they strongly convey the unpredictability and unimaginability of when and where the earth would tremble.

Today, Xevents attract both public and scientific interest, for various reasons. For example, we fear that natural Xevents could increase in frequency and intensity, possibly triggered by human activity. Furthermore, we are shaken by the sudden and abrupt collapse of structures, such as buildings, power plants, and traffic and transportation systems, which are almost always man-made and subject to further complications due to ignorance and/or negligence. It all boils down to the question of vulnerability -how can pop-ulations be protected from Xevents, especially in view of the global interdependencies of technology, economy, ecology, and society? Science can make a significant contribution in this respect, as it aims to understand the dynamics of Xevents (the processes occurring before, during and after the event); to predict the occurrence of an event and its impact; and to define the limits of prediction.

1.2 What are Xevents? A First Approach

Before we proceed any further with this discussion, we need to define exactly what we mean by an "extreme event". In the context of an extreme event, an "event" is something that happens within a limited space and time. Its occurrence can arise by chance or necessity or through a combination of both; through natural or human-made causes or a combination of both. The interpretation of "extreme" cannot be defined so easily. It encompasses a collection of attributes, such as rare, exceptional, catastrophic, surprising, and the like. An insurer would translate "rare" as "low-probability" and "catastrophic" as "of great consequence", the latter emphasising the event's potential for impact and change. Therefore, a hurricane is an Xevent only if it causes loss of life and material damage; it is considered to be an ordinary event if it hits uninhabited areas. An asteroid strike is an extreme event only when it strikes the earth and changes the course of evolution, which seems to have happened 65 1nillion years ago. The degree of "extremeness" of a Xevent, an important consideration for insurance companies, politicians and journalists, may thus be intuitively expressed as the product of the change and the impact caused by the event divided by the frequency of occurrence.

From a science viewpoint, on the other hand, the impact aspect is not the most important. What stirs scientific passion are huge deviations in a series of measurements, the burst-like nature of extremes, their apparent uniqueness: in short, the unexplainable and unpredictable. This means that the occurance of an asteroid strike is an extreme event, regardless of its impact on human life; as are magnetic storms in the magnetosphere, even if there is no recordable impact on electronic devices on earth. However, society's need to cope with the consequences of Xevents is becoming more and more urgent and so we can no longer afford to leave all Xevent-related considerations solely to policy-or decision-makers. Powerful simulation tools may help and thus we will discuss them in this book.

Xevents can also be individual, such as a first love, the birth of a child, the death of a spouse, the awarding of a Nobel Prize, to mention only a few examples. Xevents can also be general, in that they affect people and the environment: societal disasters (pandemics such as influenza and AIDS); natural disasters (floods, droughts, cyclones); technical breakdown (power outages, material ruptures, explosions, chemical contaminations); or market turbulence (huge losses or gains in the stock market), to mention but a few. World wars are undoubtedly among the most extreme of extreme events. We remind the reader of Eric Hobsdawn's fulminate *Age of Extremes* [1], which describes and analyses the social catastrophes of the twentieth century, in particular the two World Wars and the revolutions that followed each war.

The connection between wars and Xevents raises the question of morality, which quickly dominates all other issues involved: should a specific Xevent be judged as positive or negative? The wars of the twentieth century (but not only these!) are rightly considered as human tragedies unmatched in scale and consequence. However, tragedies - on whatever scale - almost always contain the *seeds* of positive change. The World Wars ultimately led to the spread of democracy around the globe (especially in Germany), ending the era of colonialism. The meteorite that is believed to have struck the earth millions of years ago extinguished the dinosaurs and facilitated the evolution of mammals; the nuclear disaster in Chernobyl, which killed thousands of people, fostered the development and implementation of alternative sustainable forms of energy; while the unification of West Germany and East Germany - widely welcomed as a positive Xevent - also gave rise to high unemployment and social displacement and deprivation. Therefore, rating Xevents as positive or negative is purely subjective; there are always trade-offs between the risks from and the benefits of the event.

1.3 What are Xevents? A Second Approach

So it seems that there are many definitions of an Xevent. This indicates that the issue is multifaceted, intricate, complex, and subject to various interpretations, perceptions, assessments, and even emotions. For science, this is not a comfortable situation. From a scientific perspective, the aim must be to free Xevents from their apparent subjectivity so that a more objective definition can be obtained. Defining a quantity (mathematical, physical, or whatever), on the other hand, requires adequate knowledge of it. This is not yet available. All we can do at the moment is to characterise Xevents by their most important elements: their statistical and dynamic properties, possible commonalities and analogies, observations, mechanisms, predictability, prediction, and management. It may be helpful to present a few remarks on these topics prior to explaining the idea underlying this volume and its components.

1.3.1 Statistical Characterisation of Xevents

From a statistical perspective, Xevents occur in the tails of probability distributions that define the occurrence of events of a given size (in terms of energy, duration, and so on). In a Gaussian distribution, these tails (situ-ated to the far left and right of the peak value) are exponentials. For many Xevents, the tails are "heavy": for instance (algebraic) power laws with some fixed power, $p(x) \alpha x^{-\alpha}, \alpha>0$. Power laws fall off much more slowly than exponential (Gaussian) distributions, indicating an enhanced probability of occurrence. We note in passing that power laws (not exponentials) possess scale invariance (corresponding to self-similarity, in terms of geometry), which is important for many natural phenomena (see below). This property can be expressed mathematically as $p(bx)=b^{-\alpha}p(x)$, meaning that the change of variable from x to bx results in a "scaling factor" independent of x, while the shape of p is conserved. So power laws represent "scale-free systems". A typical Gaussian distribution is that representing a the heights of a num-ber of people, with a well-defined mean value and a relatively small variance. Typical power laws include the distribution of wealth (known as Pareto's law, with a fraction of people presumably several times wealthier than the reader) and the size distributions of earthquakes (Gutenberg-Richter), forest fires and avalanches, among other examples.

The statistics of Xevents is known as *extreme value statistics* (this form of statistics dates back to 1958, when E.J. Gumbel published his seminal book [2]. The aim is to obtain as much information as possible on their (unknown) distribution functions. Typical problems include finding the probability that the size of an event exceeds a given value, or the largest event that will occur in a given period of time, for a given location. The assumptions used in common theories are still quite limiting, dealing mostly with independently and identically distributed events, which is rarely the case in reality. However, if the distribution function of Xevents can be estimated with sufficient accuracy, all relevant quantities (including those mentioned above) can be evaluated.

1.3.2 Dynamic Characterisation of Xevents

From what has been said so far, it appears as though Xevents are generated randomly, as with throwing dice. This is a wrong assumption. They occur in systems with complex dynamics, usually far from equilibrium, where the system's variability (not its mean values) and collective effects (not its individual aspects) are dominant. Consider weather extremes. What we call weather is the state of the Earth's atmosphere in the region relevant to us, which is continually and dynamically evolving according to well-known equations of motion (such as the Navier-Stokes equations). Therefore, modern weather prediction performed by running numerical simulations of model equations, fed by observations (measurements) as initial conditions. In fact, all natural

Xevents are almost undoubtedly phenomena that occur as manifestations of the complex dynamics of a certain system. Hence, we search for dynamic mechanisms that allow a given system to make an excursion far from its nor-mal state. Several such scenarios are known, among them the concepts called the theory of large deviations, self-organised criticality (SOC), deterministic chaos, and fully developed turbulence, to mention just a few. SOC, for instance, suggests that a system reacts to a sequence of perturbations by manoeuvring itself into a critical state (with no external tuning or organisation required) where huge fluctuations are the rule rather than the exception, and which cause power law distribution functions for the relevant observables. For instance, the aforementioned Gutenberg-Richter law of earthquake magnitude distribution can be reproduced by suitable SOC models. However, there is a wide range of potential dynamic scenarios for Xevents, some of them generating precursors, some of them requiring nonlinear positive feedback loops with evident instabilities. Hence, there is definitely no universal dynamic mechanism at work; but the Number of potential mechanisms is small. Despite a huge body of knowledge about dynamics, accumulated mainly over the past three decades, Xevents have only rarely been the focus of such studies.

1.3.3 Shaping Evolution

During the evolution of the Earth's surface, state economies, and political structures, to name three examples, Xevents have obviously had significant roles to play: they shape the future courses of such systems. Indeed, the worst earthquakes in California, with a recurrence rate of about once every two centuries, account for a significant fraction of the region's total tectonic de-formation; landscapes are changed by the "millennium" flood, which is more effective than the concerted action of all other eroding agents; the largest volcanic eruptions lead to major topographic changes and to severe climatic disruptions; financial crashes, which in an instant can cause the loss of trillions of dollars, loom and affect the psychological state of investors, society, and the world economy.

1.3.4 Commonalities, Analogies, Universality

Newton's law of gravity is universal, as it applies to any particle of matter .

Could a similar statement hold true for Xevents? Certainly not -they are too complex and too diverse. So let us modify the question: is there some (simple or complicated) mechanism that produces similarities in behaviour between different Xevents? Or will the behavior depend crucially on the specifics of each system or classes of systems, provided that such classes exist and can be defined and identified? In other words, is there any kind of universality that expresses the common nature or essence that the members of a class (individual events) share with one another? Is this universality purely formal, or can it be imagined as being dynamic and developing where universality and individuality merge?

Physicists and mathematicians are used to thinking of universality. We remind the reader of bifurcations and phase transitions (where phenomena as diverse as ferromagnetism, superconductivity, and the spread of epidemics - percolation theory - enjoy a unified theoretical description in which details of the system become irrelevant). However, as already mentioned above, we cannot expect universality. The most we can hope to find is the existence of several universality classes. Things might be more complicated than this, however, since the different facets of Xevents (origin, impact, and phenomenology) allow us to search for and to discover commonalities on different levels of description. Therefore systems falling into the same universality class when considering physical aspects might appear very disparate on the sociological level.

If there are commonalities in cause, there are many more commonalties in effect. Indeed, Xevents entail casualties: deaths, heavy financial costs, environmental destruction, and undermining the fabric of society. These result from side effects or secondary events deriving from the primary Xevent, such as the disruption of communication networks, the contamination of water, and the breakdown of health support, energy supplies, and so on.

1.3.5 Prediction, Anticipation and Management

Xevents call for *prediction.* In a sense, we believe in "savoir pour prévoir", as stated by the French philosopher Auguste Comte. Prediction implies movement from the past through to the present towards the future: a cause or several causes may lead to an effect - the extreme event - to be predicted. In a wider sense, prediction may be complemented by a proactive dimension: how do we cope with a predicted event when we don't know when it is going to occur. This requires backward travel from the future - as a possibility - to the present. *Anticipation* describes this (nonreactive) perspective. Combining prediction and anticipation is a prerequisite to managing Xevents.

Predicting may be a dangerous activity, however. There are many reasons for this. An important one is that forecast models are constructed on mean quantities. However, a real-world complex system is often better viewed as a collection of "hot spots" rather than a reasonably homogeneous background contaminated by small-scale noise. This explains why classical methods often fail. We select four striking examples from a long list of drastic failures, which were severely underestimated or not predicted at all: the storms that struck Western Europe in December 1999; heavy and devastating precipitation in Northern Italy in autumn 2000; the terrorist attacks in Madrid and Beslan (Russia) in 2004; and most recently, the Asian apocalypse in which more than 200,000 people were killed and many more made homeless on 26 December 2004 by a tsunami.

Any quantity that is to be predicted must be predictable. This depends on the quality and availability of data, existence and type of precursors, and the amount of determinism involved, such as memory effects or some depen-dence (in time or space) between the observables. The signal-to-noise ratio is an important quantity in this context. If it is low, faithful predictions are impossible. Another important quantity is the time horizon for prediction. In general, we may distinguish between short-, medium- and long-term prediction. While this holds true for ordinary time series prediction, this concept must be revised as far as Xevents are concerned. Here our desire is to predict, among other things, the largest event that will occur in a given period of time, for a given location.

However, more important than predicting an event (or more correctly, the probability of occurrence) is the specification of confidence intervals indicating the upper and lower bounds of probability. As a matter of fact, rather than trying to predict Xevents, one may define the range of all possible Xevents, just to provide the information we so desperately seek.

When we discuss consequences, we should bring up management, which refers to mechanisms used to cope with the impacts of Xevents. The effectiveness of management depends on understanding, anticipation, preparedness, and response to these events. There are avoidable (usually human-made) and unavoidable (usually natural) catastrophes. If an Xevent is avoidable, then prevention is of great importance. If it is unavoidable, then management searches for mitigation and adaptation (mathematically, this means some_ kind of optimisation); this is known as a vulnerability reduction strategy.

1.3.6 Trends

Regular structures, especially trends, can be superimposed on the irregular behaviour of a time series. So it is useful to check possible trends in the extreme values of the time series and to evaluate the extent to which they depend on technology, behaviour, habits, and so on. Some people claim that, due to human interaction, climate extremes are becoming more extreme and temperature swings occur more often. Ultimately, Xevents may become so frequent that they are no longer extreme, but define a system's norm. This conclusion is certainly very speculative, but it seems to have many adherents among the worldwide climate community.

1.3.7 Building Models

As with other scientific problems the modelling approach is the most effective and appropriate one to apply to Xevents. One may roughly differentiate between diagnostic models, which investigate *what has happened,* and prognostic models, which investigate *what will happen.* These (microscopic or macroscopic, general or specific) models are always some simplification of reality, which is usually reduced to some basic physical laws and formalised in mathematical terms. Computer simulations also play an important role. Simulations are used to answer question like "Suppose some relevant parameter is changed, what is the response of the system if all other factors are kept constant?" In order to obtain valid answers; the current state of the system must be properly reproduced by the simulation, which requires the optimisation of system parameters and functions. This is difficult to achieve in most cases, which sheds some doubt on simulations in general. Dynamic models contain nonlinear feedback, and the solutions to these are usually obtained by numerical methods. Statistical models are data driven; in their simplest version they try to fit a given set, of data using various techniques. There are hybrids, coupling dynamic and statistical aspects, including deterministic and stochastic elements. Simulations are often based on cellular automata and network formalisms, connecting input and output in nonlinear ways. These models are calibrated by training the networks, so that the error between output and given test data is minimised.

1.3.8 Observations

The underlying reality of theories is data. In fact, observations constitute a firm base from which scientific reasoning can start and to which it must always return in order to test its validity. So without data - and this is a possible way of thinking - there is no theory, or at least no verifiable theory. In our context, this means a sequence of data forming a time series, in which a measurement point is associated with each time point (often equally spaced). Collecting data, organising data and drawing conclusions (statistical inference) is achieved through ordinary data handling. For Xevents, it is not the mean value that matters, but rather the deviations from this, in particular the greatest deviations. A distribution function of the extreme values is required. A special problem arises here which is sometimes called the "curse of few observations", meaning that Xevents tend to be rare and thus impede meaningful statistical inference. The lack of observations is, in many cases, overcome by using extreme value statistics (see above).

1.3.9 Risk

For an individual, "risk" means the probability of an undesired outcome, such as disease or death, resulting from bad habits or an unfavourable environment, among other possible causes. For finance, risk is the uncertainty that the actual return of an investment will be less than the expected one; due to inflation or fluctuating currency -exchange rates, for example.

Sociologists go beyond the microscopic view of risk. Risk becomes a macroscopic or social phenomenon. Ulrich Beck received much attention when he created his "risk society" [3]. He observed a transition from "old" society - whose fate was determined by naturally occurring hazards along with socially induced hazards, such as wars -to a "modern" one, whose course is governed by risks, especially those driven by industry and new technology. According to Beck, risk, rather than social deprivation or inequality, is the constituent ele-ment of society, causing new conflicts and social formations. A "risk society" seems to entail an "insurance society", in which damage is compensated by money. For insurers and re-insurers, risk is a measure of uncertainty ranging between 0 (highly uncertain) and 1 (certainty), proportional to the product of probability of occurrence and damage. Risk can be insured if it is computable and identifiable. However, this is not always possible, since Xevents are -as has already been stated above -difficult to estimate, both in cause and in effect; in addition, they can be accompanied by enormous and highly correlated losses. Other forms of protection, such as prevention or precaution, must be developed in order to "tame" Xevents.